Experiments on the onset of wave formation on a film of water flowing down a vertical plane

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SUMMARY

The supply of water to the outside of a long vertical tube was adjusted until the water film was seen to be only just disturbed by a train of travelling waves. Under these conditions the Reynolds number, defined as the discharge per unit width divided by the kinematic viscosity, was 4·4. The wave train was slightly irregular, and average values of the length and velocity of the waves were about 0·45 in. and $5\frac{1}{2}$ in./sec, the temperature being 19° C.

1. INTRODUCTION

When a thin sheet of water passes over a smooth inclined plane, such as a slab of rock or a tarred road, its surface is sometimes disturbed by travelling waves even in the absence of wind. These roll waves, as they are often termed, occur with their crests normal to the direction of streaming, and photographs of those produced in highly turbulent form in an embanked torrent of uniform slope and cross-section were published by Cornish (1934, plates 21–23). Under laboratory conditions in still air numerous experiments have been made, mostly on vertical surfaces, and the observations can be recorded in terms of the Reynolds number R defined by $R = Q/\nu$, where Q is the discharge per unit width of the stream and ν is the kinematic viscosity of the liquid. If the slope is vertical and the liquid is water, the film to be examined is very thin and the experimental difficulties are considerable. In vertical flow the motion is not obviously turbulent below about $R = 250$, and over most of this range the surface of the liquid is agitated by irregular ripples, single photographs of which were published by Grimley (1945). Dukler & Bergelin (1952), after giving a detailed survey of the experimental literature, reported their measurements of ripple profiles at Reynolds numbers down to about 196 which showed steep-fronted slopes with heights that exceeded the mean film thickness. But most of the investigations have been concerned with finding the minimum Reynolds number at which waves can be observed. The results show a marked scatter. Friedman & Miller (1941) gave $R = 6{\cdot}25$ as the limit, and Grimley's value (1945) was 6·2, but his photograph obtained at $R = 6{\cdot}65$ showed waves that were apparently rather large as well as irregular. On the other hand Kirkbride's result (1934) was 2, In the next section an account is given of an attempt to determine not only the smallest Reynolds number at which waves could be seen under good conditions of illumination but also the velocity and length of these waves. The liquid used was water, which was caused to flow down the outside of a long vertical glass tube. The thickness of the film was so small in comparison with the radius of the tube that the motion could be taken to be the same as that down a plane surface.

2. Description of experiments

The apparatus, which is shown in diagrammatic elevation in figure 1 (plate 1), was built round a fixed 5 ft. length of Truebore glass pipe of outside diameter 1·045 in. Distilled water passed from an elevated reservoir through a control valve and two alternative Rotameters into the upper Perspex chamber. A hole in the thick base of this chamber was slightly larger than the glass tube, and it could be centred by four adjusting screws threaded in the wall and pressing against the glass tube. In this way the water was delivered to the outside of the tube in as uniform a stream as possible. At the bottom the water was caught in another chamber and fell into a lower reservoir whence it was returned to the upper by a pump. The two chambers were joined by a Perspex pipe, held in position by rubber tubing. The pipe served as a dust cover to the working section, and in it a large window was cut which was removed only when photography was actually in progress. Four sprays were added at the extreme top of the working section so that, when required, the surface of the working section could be thoroughly wetted. They were supplied from a branch on the delivery side of the pump. For observation by eye, illumination at about 30° incidence at the middle of the working section was provided by bulbs and reflectors. A closely-fitting cylinder of glossy white paper was pushed down the inside of the glass tube; and when the waves were small, only their shadows on this paper were visible. The wave system extended all down the tube, and the motion appeared to be independent of the distance from the top. In spite of the care taken with cleaning and with precautions against the entry of dust, much experience with the apparatus was required before it would operate consistently at very low velocities.

The whole of the photographic work was very kindly carried out by Dr R. H. J. Brown, who used the repetitive flash apparatus that he has already described in detail (1952). Here it is sufficient to state that the flash duration of the lamp was about 10 microseconds, and that up to 95 pictures per second could be taken; thus the range of his apparatus was amply sufficient to meet the requirements of the present problem. The focused beam from the lamp was directed upwards at about 20° incidence, and it also illuminated a clock and a vertical scale placed close to the working section, which was photographed horizontally.

The procedure, when the final photographs were taken, was as follows. After the apparatus had been running at a moderate velocity for half an hour, the pump was stopped and the discharge slowly reduced while the tube

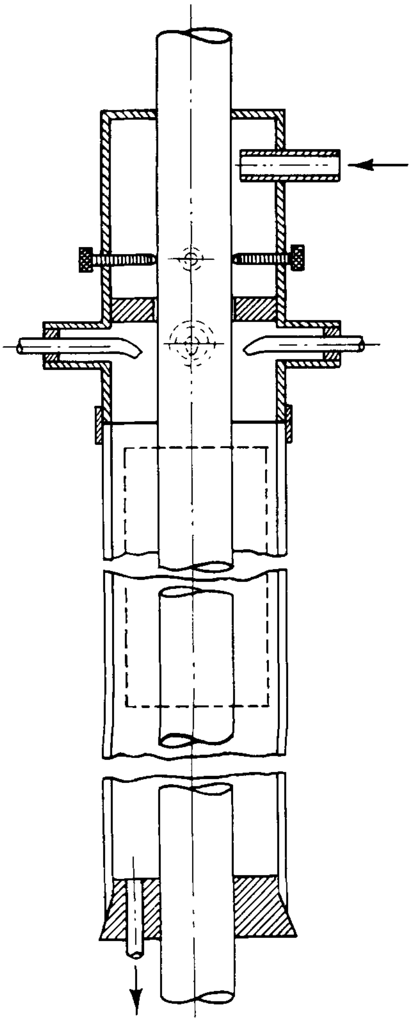

Figure 1. Elevation of apparatus.

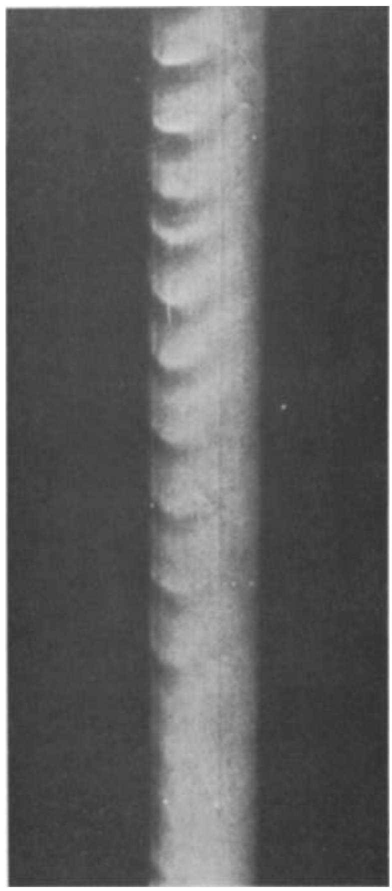

Figure 2. Shadow of limiting train of waves.

was repeatedly re-centred to produce waves of apparently the same amplitude all round the periphery. At the lowest velocity tried, waves could barely be seen on the front of the tube and over some but not all of the remainder of the periphery. The discharge was then very slightly increased, causing waves all round; and thus it was checked that the waveless zone had not dried off. When the velocity was again reduced to its previous value, it was seen that the wave pattern was the same as before. After this process of raising and lowering the flow had been carried out several times, it was thought that the minimum condition had been attained. The eye could just perceive a continuous stream of waves with horizontal crests, all moving with the same velocity but having slightly different lengths. Occasionally a larger and faster wave appeared that passed through the procession without permanently disturbing it. Two bursts of exposures were then taken and the flow determined by diverting and weighing the discharge from the base of the apparatus. The discharge per inch width of the tube circumference was $6{\cdot}9 \times 10^{-3}$ in.^{3}/sec at a temperature 19° C, thus $R = 6{\cdot}9/1{\cdot}58 = 4{\cdot}4$.

Eighty-three pictures were obtained at intervals of 0·047 sec, and they were examined with the aid of a projector. The specimen, reproduced in figure 2 (plate 1), shows one of the most regular wave trains that were photographed. Measurements were confined to the top 3 in. of the field of view, about 12 in. below the start of the working section, because there the shadows were most sharply defined and the distortion due to lack of parallelism in the light beam was least. The progress of typical single waves could be traced without difficulty on successive pictures because of the slight irregularities in the wave spacing, and recognition was helped by small density variations in some of the shadows, which suggested departure from the simple sinusoidal profile. Five crests followed in this way were found to travel 1·57, 1·53, 1·62 in. in 6 frames and 1·83, 1·87 in. in 7 frames, giving velocities 5·6, 5·4, 5·7, 5·6, 5·7 in./sec. In one picture all the waves had lengths between 0·45 and 0·5 in., except for one that was only 0·31 in. from its follower. A train of four equally spaced crests, occupying a distance 1·30 in., moved 1·60 in. in 6 frames, thus the wavelength was 0·43 in. and the velocity 5·7 in./sec. Average values of the length and velocity were therefore about 0·45 in. and $5\frac{1}{2}$ in./sec.

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